

PART I. Solve the following problems. Show your solutions in detail.

1. Three point charges are located at the vertices of a square of side $a = 5.0 \text{ cm}$, as shown in the figure. Compute the net force exerted on the point charge $q = 5.0 \text{ nC}$. Given $q_1 = 3.0 \text{ nC}$, $q_2 = 7.0 \text{ nC}$, and $q_3 = 6.0 \text{ nC}$. [4 points]

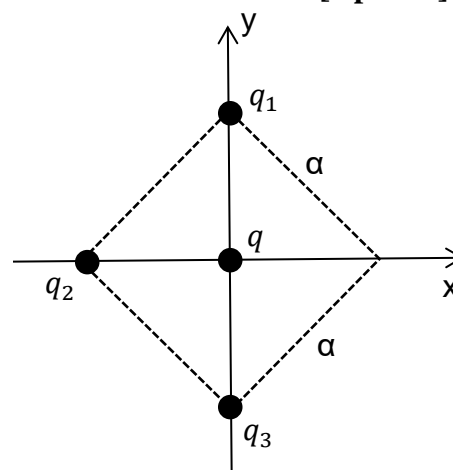
$$a^2 = r^2 + r^2 \Rightarrow r^2 = a^2/2$$

$$\vec{F}_1 = k \frac{q_1 q}{\left(\frac{a^2}{2}\right)} (-\hat{j}) \Rightarrow \vec{F}_1 = 1.08 \times 10^{-4} \text{ N} (-\hat{j})$$

$$\vec{F}_2 = k \frac{q_2 q}{\left(\frac{a^2}{2}\right)} (+\hat{i}) \Rightarrow \vec{F}_2 = 2.52 \times 10^{-4} \text{ N } \hat{i}$$

$$\vec{F}_3 = k \frac{q_3 q}{\left(\frac{a^2}{2}\right)} (+\hat{j}) \Rightarrow \vec{F}_3 = 2.16 \times 10^{-4} \text{ N } \hat{j}$$

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 \Rightarrow \vec{F}_{net} = 2.52 \times 10^{-4} \text{ N } \hat{i} + 1.08 \times 10^{-4} \text{ N } \hat{j}$$

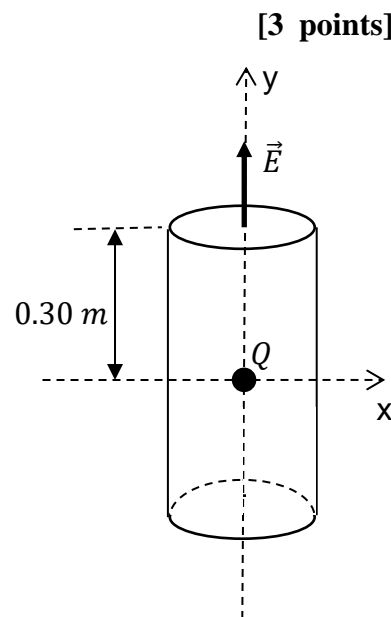


2. A point charge Q is placed in a cylinder as shown in the figure. The electric field at the center of the top of the cylinder is $4 \times 10^3 \text{ N/C}$, as shown in the figure. Find the flux through the cylinder. [3 points]

$$E = k \frac{Q}{(0.30)^2}$$

$$Q = \frac{(0.30)^2 E}{k} \Rightarrow Q = 40 \text{ nC}$$

$$\Phi = \frac{Q}{\epsilon_0} \Rightarrow \Phi = 4520 \text{ Nm}^2/\text{C}$$



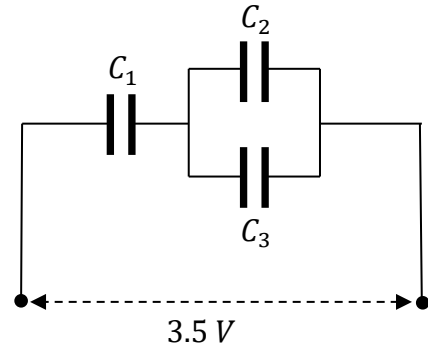
3. In the figure below, the charge on capacitor C_2 is $15 \mu C$. Find the charge on capacitor C_3 . Given $C_1 = C_2 = 10 \mu F$. [4 points]

$$V_2 = \frac{Q_2}{C_2} \Rightarrow V_2 = V_3 = V_{23} = 1.5 V$$

$$3.5 V = V_1 + V_{23} \Rightarrow V_1 = 2.0 V$$

$$Q_1 = C_1 V_1 \Rightarrow Q_1 = 20 \mu C$$

$$Q_3 = Q_{23} - Q_2 \Rightarrow Q_3 = Q_1 - Q_2 = 5 \mu C$$



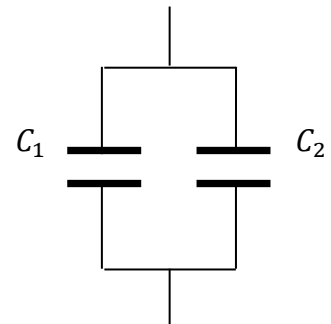
4. A fully charged capacitor $C_1 = 5 \mu F$ is connected to an uncharged capacitor $C_2 = 10 \mu F$, as shown in the figure. If the final energy stored in C_2 is $180 \mu J$, find the initial energy stored in C_1 before it was connected to capacitor C_2 . [4 points]

$$U_2 = \frac{1}{2} C_2 V_2^2 \Rightarrow V_2 = \sqrt{\frac{2U_2}{C_2}} = 6.0 V$$

$$C_{12} = C_1 + C_2 \Rightarrow C_{12} = 15 \mu F$$

$$Q_{total} = C_{12} V_2 \Rightarrow Q_{total} = 90 \mu C$$

$$U_1 = \frac{1}{2} \frac{1}{C_1} Q_{total}^2 \Rightarrow U_1 = 810 \mu J$$



5. A uniform 1.0 m long steel wire is connected to a 12 V battery. The free electrons run the length of the steel wire in 250 s and the concentration of free electrons in steel is $9.0 \times 10^{28}\text{ electrons/m}^3$. Find the resistivity of the steel. **[3 points]**

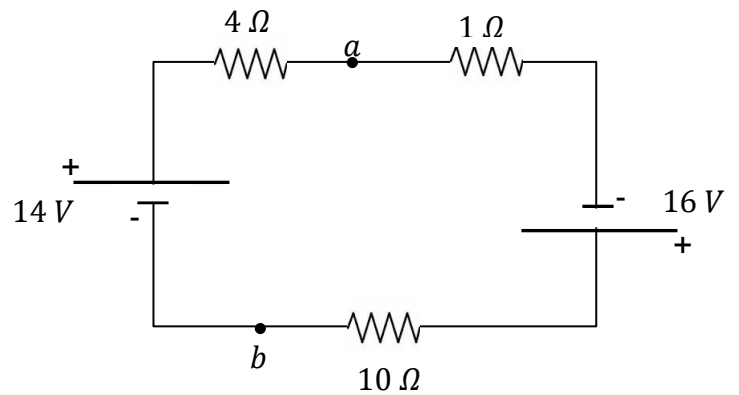
$$v_d = \frac{L}{t} \Rightarrow v_d = 4 \times 10^{-3}\text{ m/s}$$

$$V = IR \Rightarrow V = n|e|v_d A \rho \frac{L}{A} = n|e|v_d \rho L$$

$$\rho = \frac{V}{n|e|v_d L} \Rightarrow \rho = 20.8 \times 10^{-8}\ \Omega\text{m}$$

6. In the circuit shown in the figure, find the potential difference V_{ab} .

[2 points]



Loop rule:

$$14 - 4I - 1I + 16 - 10I = 0 \Rightarrow I = 2\text{ A}$$

$$V_a - 1I + 16 - 10I = V_b \Rightarrow V_{ab} = -16 + 11I \Rightarrow V_{ab} = 6\text{ V}$$

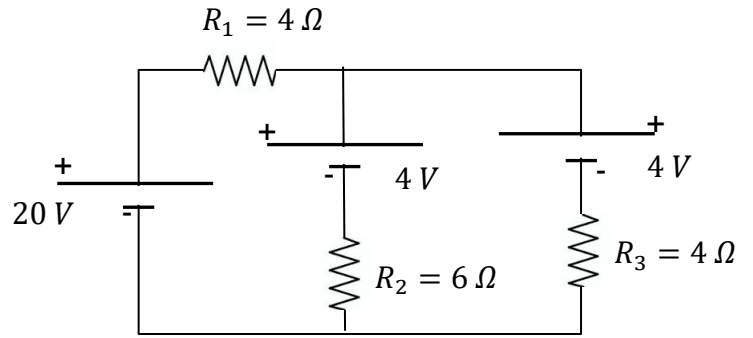
7. The power delivered in the circuit below by the 20 V battery is 50.0 W . Find the current in the $R_3 = 4.0\ \Omega$ resistor. **[3 points]**

$$P = VI \Rightarrow I = \frac{P}{V} \Rightarrow I = 2.5\text{ A}$$

Big Loop:

$$20 - 4I - 4 - 4I_2 = 0$$

$$I_2 = 1.5\text{ A}$$



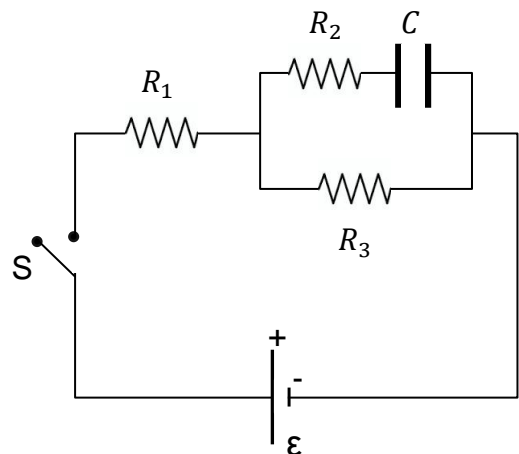
8. In the circuit below, $R_1 = 10.0\ \Omega$, $R_2 = 15.0\ \Omega$, $R_3 = 20.0\ \Omega$, and $\varepsilon = 60.0\text{ V}$. If the switch S is closed at $t = 0\text{ s}$, find the charge on the capacitor of capacitance $C = 2\text{ nF}$ after a very long time. **[3 points]**

Loop rule:

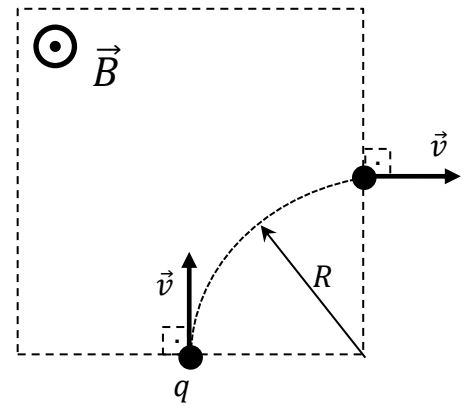
$$60\text{ V} - IR_1 - IR_3 = 0 \Rightarrow I = 2\text{ A}$$

$$V_c = V_3 = IR_3 \Rightarrow V_c = 40\text{ V}$$

$$Q = CV_3 \Rightarrow Q = 80\text{ nC}$$



9. A positive point charge q enters perpendicularly a uniform magnetic field with velocity \vec{v} as shown in the figure. The point charge exits perpendicularly the magnetic field after time $t = 4\pi \times 10^{-5} \text{ s}$. Find the speed of the point charge. Given $R = 20.0 \text{ cm}$ [3 points]



$$R = \frac{mv}{Bq}$$

$$t = \frac{T}{4} = \frac{1}{4} \frac{2\pi m}{Bq} \Rightarrow t = \frac{\pi m}{2Bq}$$

$$\frac{R}{t} = \frac{2v}{\pi} \Rightarrow v = \frac{\pi R}{2t}$$

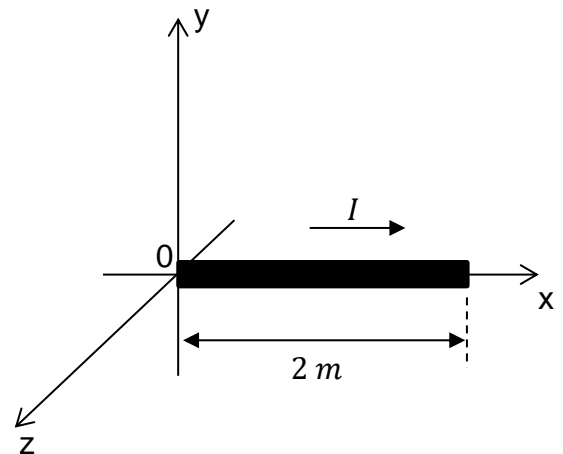
$$v = 2500 \text{ m/s}$$

10. The 2 m long wire shown in the figure carries a current $I = 5 \text{ A}$ and is placed in a magnetic field $\vec{B} = (4\hat{i} - 3\hat{j} + 5\hat{k})\text{ T}$. Find the magnetic force acting on the wire. [3 points]

$$\vec{F} = I\vec{L} \times \vec{B}$$

$$\vec{F} = 5 \times \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 0 \\ 4 & -3 & 5 \end{vmatrix} \text{ N}$$

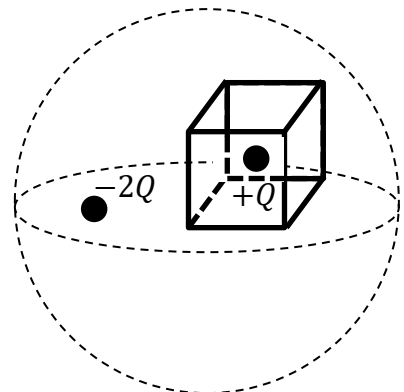
$$\vec{F} = -50\text{ N } \hat{j} - 30\text{ N } \hat{k}$$



Part II. Multiple Choice Questions (each carries 1 point). Tick the best answer.

1. Two point charges, q_1 and q_2 , are a distance d apart. The force on q_1 increases by a factor of nine (9), if
- a. the distance between, q_1 and q_2 is increased by a factor of nine (9).
 - b. the distance between, q_1 and q_2 is increased by a factor of three (3).
 - c. the distance between, q_1 and q_2 is decreased by a factor of nine (9).
 - d. the distance between, q_1 and q_2 is decreased by a factor of three (3).
2. A conductor with a cavity carries a net charge Q and then a point charge q is placed in the cavity. The magnitude of the electric field in the cavity (not at the location where the point charge is placed) and at a distance r from charge q is given by the expression
- a. $k \frac{Q}{r^2}$
 - b. $k \frac{q}{r^2}$
 - c. $k \frac{(Q+q)}{r^2}$
 - d. $k \frac{(Q-q)}{r^2}$
3. Two infinitely large parallel sheets a distance d apart carry equal but opposite uniform surface charge densities. A point charge that is placed midway feels a force F due to the two sheets. If this point charge is now moved closer to one of the sheets, so that it is a distance $d/4$ from that sheet, what force will the point charge feel?
- a. $F/4$
 - b. $F/2$
 - c. F
 - d. $2F$
4. A point charge Q is placed inside a cube. A Gaussian sphere encloses the cube and a point charge $-2Q$ which is placed inside the sphere but outside the cube. If Φ_C is the flux through the cube while Φ_S is the flux through the sphere, then

- a. $\Phi_S = +\frac{Q}{\epsilon_0}$ and $\Phi_C = +\frac{Q}{\epsilon_0}$
- b. $\Phi_S = +\frac{Q}{\epsilon_0}$ and $\Phi_C = -\frac{Q}{\epsilon_0}$
- c. $\Phi_S = -\frac{Q}{\epsilon_0}$ and $\Phi_C = -\frac{Q}{\epsilon_0}$
- d. $\Phi_S = -\frac{Q}{\epsilon_0}$ and $\Phi_C = +\frac{Q}{\epsilon_0}$

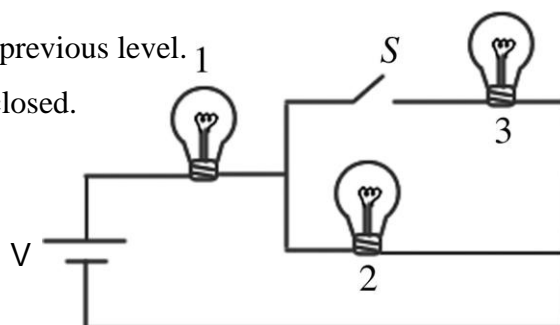


5. A capacitor is connected to a battery and has a charge Q when it is fully filled with a material of dielectric constant κ . If the battery remains connected but the dielectric material is taken out, what will be the potential energy of the capacitor?

- a. $U = \frac{1}{2} QV$
- b. $U = \frac{1}{2} \kappa QV$
- c. $U = \frac{1}{2\kappa} QV$
- d. $U = 0$

6. The figure shows three identical lightbulbs connected to a battery having constant voltage V across its terminals. What happens to the brightness of lightbulb 1 when the switch S is closed?

- a. The brightness increases momentarily, then returns to its previous level.
- b. The brightness remains the same as before the switch is closed.
- c. The brightness decreases permanently.
- d. The brightness increases permanently.



7. A charged particle is moving with a speed v perpendicular to a uniform magnetic field. A second identical charged particle is moving with speed $4v$ perpendicular to the same magnetic field. If the angular speed of the first particle is ω , the angular speed of the second particle is

- a. 2ω
- b. ω
- c. $\omega/2$
- d. $\omega/4$

8. A current-carrying wire with sides of length L and electric current I is placed in a uniform magnetic field \vec{B} as shown in the figure. The magnitude of the magnetic force exerted on the wire is

- a. $F=3BIL$
- b. $F=2BIL$
- c. $F=BIL$
- d. $F=0$

